

**Language Services**

**TRANSLATION**

**DIN 3960 Supplement 1**

**July 1980**

**Definitions on involute  
cylindrical gears and  
gear pairs,  
classification of the equations**

**original language version**  
**Begriffe und Bestimmungsgrößen für Stirnräder (Zylinderräder) und Stirnradpaare (Zylinderadpaare)**  
**mit Evolventenverzahnung. Zusammenstellung der Gleichungen.**

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Definitions on involute  
cylindrical  
gears and gear pairs,  
classification  
of the equations

Supplement 1  
to  
DIN 3960

German title:  
*Begriffe und Bestimmungsgrößen für Stirnräder  
(Zylinderräder) und Stirnradpaare (Zylinderradpaare)  
mit Evolventenverzahnung  
Zusammenstellung der Gleichungen*

This Supplement contains information  
relevant to DIN 3960, but does not contain  
any additional standard specifications.

This Supplement 1 to DIN 3960 contains the equations from DIN 3960, July 1980 issue. These equations are arranged in alphabetical order according to the head words.

For application limits to be observed for some equations refer to DIN 3960.

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**1 SYMBOLS AND DESIGNATIONS FROM DIN 3960**

**1.1 Symbols and designations**

The following symbols and designations are used in DIN 3960:

- $\alpha$  Centre distance
- $\alpha_d$  Zero centre distance
- $\alpha''$  Double flank pitch distance
- $b$  Facewidth
- $b_M$  Contact line overlap or measuring surface width for facewidth measurements
- $c$  Bottom clearance
- $c^*$  Bottom clearance factor
- $d$  Reference circle diameter
- $d_a$  Tip circle diameter
- $d_{Am}$  Measured value of tip circle diameter for overlapping cylindrical gears
- $d_b$  Base diameter
- $d_f$  Root circle diameter
- $d_n$  Equivalent reference circle diameter
- $d_v$  V-circle diameter
- $d_w$  Pitch circle diameter
- $d_y$  Y-circle diameter
- $d_K$  Diameter of ball centre point circle
- $d_M$  Measuring circle diameter (at contact point with measuring equipment)
- $d_N$  Effective circle diameter
- $d_{Na}$  Effective tip circle diameter
- $d_{Nf}$  Effective root circle diameter
- $d_{NfE}$  Root circle diameter on generated wheel (usable tooth flank)
- $e$  Space width on reference cylinder
- $e_a$  Space width on tip cylinder
- $e_b$  Base space width (on the base cylinder)

$e_f$	Space width on the root cylinder
$e_v$	Space width on the V-cylinder
$e_y$	Space width on the Y-cylinder
$f$	Individual deviation
$f_b$	Base circle deviation
$f_e$	Eccentricity
$f_f$	Profile shape deviation
$f_{fw}$	Profile waviness
$f_i'$	Tangential tooth-to-tooth composite error
$f_i''$	Radial tooth-to-tooth composite error
$f_p$	Individual pitch deviation
$f_{pe}$	Base pitch deviation
$f_{px}$	Axial pitch deviation
$f_{pz}$	Helix slope deviation
$f'_p$	Individual pitch deviation
$f_r$	Tip circle runout
$f_u$	Pitch spread
$f_{Ef}$	Generating shape deviation
$f_{HE}$	Generating angular deviation
$f_{H\alpha}$	Profile angular deviation
$f_{HB}$	Mesh line angular deviation
$f_\alpha$	Mesh angle deviation
$f_\beta$	Helix angle deviation
$f_{\beta f}$	Mesh angle shape deviation
$f_{\beta w}$	Mesh angle waviness
$f_{\Sigma B}$	Axial offset
$f_{\Sigma \delta}$	Axial slope
$g$	Path of contact
$g_a$	Length of exit contact path

$g_f$	Length of entry contact path
$g_\alpha$	Length of contact path (total)
$g_{\alpha a}$	Length of tip contact path
$g_{\alpha f}$	Length of root contact path
$g_{\alpha y}$	Distance of point Y from pitch point C
$g_\beta$	Ratio
$h$	Tooth depth (between tip and root line)
$h_a$	Addendum
$h_f$	Dedendum
$h_w$	Common tooth depth
$h_p$	Tooth depth of basic tooth profile
$\bar{h}_a$	Height over the chord $\bar{S}_n$
$\bar{h}_c$	Height over the constant chord
$i$	Transmission ratio
$inv$	Involute function
$j$	Backlash
$j_n$	Normal backlash
$j_r$	Radial backlash
$j_t$	Torsional backlash
$k$	Addendum modification factor
$k$	Number of teeth measured for measurement of facewidth
$k$	Number of teeth or divisions in one area
$m$	Module
$m_b$	Basic module
$n$	Speed (rotational frequency)
$n_a$	Speed (rotational frequency) of driving gear
$n_b$	Speed (rotational frequency) of driven gear
$p$	Pitch on reference cylinder
$p_b$	Pitch on base cylinder

$p_e$	Mesh pitch
$p_k$	Circular pitch (total pitch)
$p_v$	Pitch on the V-cylinder
$p_x$	Axial pitch
$p_y$	Pitch on the Y-cylinder
$p_z$	Helix slope
$r$	Reference radius
$r_a$	Tip radius
$r_b$	Base radius
$r_f$	Root radius
$r_v$	V-circle radius
$r_w$	Pitch circle radius
$r_y$	Y-circle radius
$s$	Tooth thickness on the reference cylinder
$s_a$	Tooth thickness on the pitch cylinder
$s_b$	Basic tooth thickness (on the base cylinder)
$s_v$	Tooth thickness on the V-cylinder
$s_w$	Tooth thickness on the pitch cylinder
$s_y$	Tooth thickness on the Y-cylinder
$\bar{s}$	Chordal tooth thickness
$\bar{s}_c$	Constant chord
$u$	Tooth number ratio
$v$	Linear velocity
$v_g$	Running speed
$v_{ga}$	Running speed on tooth tip
$v_{gf}$	Running speed on tooth root
$v_t$	Circumferential speed, tangential speed
$x$	Addendum modification factor
$x_{\min}$	Addendum modification factor at undercut limit

$x_E$	Generation addendum modification factor
$x''$	Profile modification factor for radial tooth-to-tooth contact
$y$	Reference circle clearance factor
$z$	Number of teeth <sup>1)</sup>
$z_a$	Number of teeth on driving wheel
$z_b$	Number of teeth on driven wheel
$z_{nx}$	Virtual number of teeth for profile modification calculations
$z_{nM}$	Virtual number of teeth for ball or roller dimensions
$z_{nW}$	Virtual number of teeth for facewidth calculations
$A$	Deviation
$A_a$	Centre distance deviation
$A_{a''}$	Deviation of radial tooth-to-tooth clearance
$A_{da}$	Tip circle deviation for overlapping spur gears
$A_j$	Tooth thickness deviation (on the reference cylinder)
$A_{sy}$	Tooth thickness deviation on the $\gamma$ -cylinder
$A_s$	Deviation of tooth chordal thickness
$A_{Md}$	Deviation of diametric two-ball or two-roller dimension
$A_{Mr}$	Dimension of radial single-ball or single-roller dimension
$A_w$	Base tangent length error
$A$	Starting point of contact
$B$	Internal simple contact point on driving wheel
$C$	Pitch point
$D_M$	Diameter of measuring ball or roller
$D$	External simple contact point on driving wheel
$E$	Final point of contact
$F$	Total deviation, composite deviation
$F_f$	Total profile deviation
$F_i$	Single-face composite deviation

$F_{i''}$	Two-face composite deviation
$F_p$	Pitch total deviation
$F_{pk}$	Pitch circumferential deviation over k pitches
$F_{pz/8}$	Pitch circumferential deviation over 1/8 of wheel circumference
$F_{\cdot p}$	Composite total pitch deviation
$F_r$	Runout of a tooth
$F_{rR}$	Runout on rear face of wheel
$F_{rV}$	Runout on front face of wheel
$F''_r$	Rolling out-of-true
$F_E$	Total generating deviation
$F_\beta$	Total mesh line deviation
$F_o$	Wobble
$K_g$	Sliding factor
$K_{ga}$	Sliding factor on tooth tip
$K_{gf}$	Sliding factor on tooth root
$L$	Measuring point spacing
$L_a$	Composite length from origin of involution to tooth tip
$L_f$	Composite length from origin of involution to tooth root
$L_y$	Composite length to point Y
$L_E$	Generating test range
$L_G$	Bearing centre spacing at a wheel axis
$L_\alpha$	Profile test range
$L_\beta$	Mesh line test range
$M_{dK}$	Diametral two-ball dimension
$M_{dR}$	Diametral two-roller dimension
$M_{rK}$	Radial single-ball dimension
$M_{rR}$	Radial single-roller dimension
$P$	Contact point (e.g. between measuring ball and tooth face)
$R$	Variation

$R_j$	Backlash variation
$R_p$	Pitch variation
$R_s^-$	Tooth thickness variation
$R_s$	Chordal tooth thickness variation
$R_{Md}$	Variation in diametral two-ball or two-roller dimension
$R_{Mr}$	Variation in radial single-ball or single-roller dimension
$R_w$	Base tangent length variation
T	Tolerance
$T_a$	Centre distance tolerance
$T_{a''}$	Tolerance of tooth-to-tooth rolling clearance
$T_{da}$	Tip circle diameter tolerance for overlapping spur gears
$T_s$	Tooth thickness tolerance
$T_s$	Tolerance of chordal tooth thickness
$T_{Md}$	Tolerance of diametral two-ball or two-roller dimension
$T_{Mr}$	Tolerance of radial single-ball or single-roller dimension
$T_w$	Base tangent length tolerance
$W_k$	Base tangent length over k measuring teeth or measuring gaps
Y	Any point on a tooth face
$\alpha$	Mesh angle = profile angle on reference cylinder
$\alpha_a$	Profile angle on tip cylinder
$\alpha_v$	Profile angle on V-cylinder
$\alpha_w$	Profile angle on pitch cylinder
$\alpha_{wt}$	Operating contact angle
$\alpha_y$	Profile angle on Y-cylinder
$\alpha_K$	Profile angle on ball centre-point circle
$\alpha_M$	Profile angle on measuring circle
$\alpha''$	Contact angle for two-flank rolling test using gauge gear wheel
$\beta$	Helix angle on reference cylinder

$\beta_b$	Basic helix angle
$\beta_v$	Helix angle on V-cylinder
$\beta_w$	Helix angle on pitch cylinder
$\beta_y$	Helix angle on Y-cylinder
$\beta_M$	Helix angle on measuring circle
$\gamma$	Helix angle on reference cylinder
$\gamma_b$	Basic pitch angle
$\epsilon$	Overlap
$\epsilon_\alpha$	Addendum overlap
$\epsilon_\beta$	Overlap ratio
$\epsilon_\gamma$	Total overlap
$\zeta$	Specific slip
$\zeta_f$	Specific slip at the end point of the contact path
$n_v$	Tooth spaces - half angle on reference circle
$n_b$	Basic spaces - half angle
$n_f$	Tooth spaces - half angle on route circle
$n_v$	Tooth spaces - half angle on V-circle
$n_w$	Tooth spaces - half angle on pitch circle
$n_y$	Tooth spaces - half angle on Y-circle
$\zeta$	Pitch angle of involute
$\zeta_a$	Pitch angle of involute on end of tooth tip
$\zeta_f$	Pitch angle of involute on end of tooth root
$\zeta_y$	Pitch angle of involute in point Y
$Q$	Radius of curvature
$Q_{an}$	Radius of curvature of tip edges in spur gear normal plane
$Q_{a0}$	Radius of tip edge curvature on tool
$Q_y$	Radius of curvature of involute in point Y
$\tau$	Reference angle

1) The number of teeth of an externally geared wheel is positively calculated, the number of teeth of an internally-geared wheel negatively. Refer to DIN 3960 Clause 2.1.

$\phi$	Centre angle, overlap angle
$\phi_c$	Centre angle between the maximum values of runouts $F_{rv}$ and $F_{rR}$
$\phi_a$	Addendum overlap angle
$\phi_B$	Overlap angle ratio
$\phi_Y$	Total overlap angle
$\psi$	Tooth thickness half angle on reference circle
$\psi_a$	Total thickness half angle on tip circle
$\psi_b$	Base thickness half angle
$\psi_n$	Virtual tooth thickness half angle
$\psi_v$	Tooth thickness half angle on V-circle
$\psi_w$	Tooth thickness half angle on pitch circle
$\psi_Y$	Tooth thickness half angle on Y circle
$\omega$	Angular velocity
$\omega_a$	Velocity of driving wheel
$\omega_b$	Velocity of driven wheel
$\Delta$	Difference
$\Delta W$	Length difference in measurement of facewidth
$\Delta\phi$	Rotational angle differences
$\Sigma x$	Sum of addendum modification factors
$\Sigma z$	Sum of number of teeth

Further abbreviations for gears are given in DIN 3999. The following standards also give details of symbols:

DIN 1302	Mathematical symbols
DIN 1304	General formula symbols
DIN 1313	Physical variables and equations; definitions, notation
DIN 66030	Presentation of unit names in systems with a limited character set (at present draft).

## 1.2 Additional indices

The following additional indices may occur in addition to the symbols in Clause 1.1:

- a for the driving wheel
- b for the driven wheel
- e for the upper limit of a variable
- i for the lower limit of a variable
- l for "left hand" or "in the sense of a left hand screw"
- m for a mean value
- n for variables in a normal plane
- r for "right hand" or "in the sense of a right hand screw"
- t for variables in a transverse plane or tangential direction
- x for variables in an axial plane or in a direction of the axis of the wheel
- L to designate left hand flanks or a master gearwheel
- M to designate a measured value
- P for variables of the basic tooth profile
- R for designation of right hand flanks
- max for a maximum value
- min for a minimum value
- zul to designate a permissible limit
- 0 for variables on the generating tool
- 1 for variables on the small wheel of a wheel pair
- 2 for variables on the large wheel of a wheel pair
- \* for designation of a factor by means of which a variable in parts or multiples of the standard module (e.g.  $c = c^* \cdot m$  or  $h_{ap} = h^*_{ap} \cdot m_n$ ) or the number of teeth (e.g.  $z^*_{nx}$  or  $z^*_{nM}$  or  $z^*_{nW}$ ) is expressed, or for designation of a deviating factor.

## 2 Summary or equations

The numbers given in brackets after the equations are the equation numbers from DIN 3960, July 1980 issue.

**Deviations:** refer to tooth thickness deviations

### Deviation factors

$$A_{a^*}^* = \frac{A_{a^*}}{A_s} \approx \frac{1}{2} \cdot \frac{\cos \alpha_t}{\sin \alpha'' \cdot \cos \beta} \quad (5.5)$$

$$A_{da}^* = \frac{A_{da}}{A_s} = \cot \alpha_n \quad (5.6)$$

$$A_{Sv}^* = \frac{A_{Sv}}{A_s} \approx \frac{d_v \cdot \cos \beta_v}{d \cdot \cos \beta} \times (\cos \psi_v - \sin \psi_v \cdot \tan(\alpha_{vt} - \psi_v)) \quad (5.1)$$

$$A_{Mid}^* = \frac{A_{Md}}{A_s} \approx \frac{\cos \alpha_t}{\sin \alpha_{kt} \cdot \cos \beta} \quad (5.3)$$

$$A_{Mid}^* = \frac{A_{Md}}{A_s} \approx \frac{\cos \alpha_t}{\sin \alpha_{kt} \cdot \cos \beta} \cdot \cos \frac{\pi}{2 \cdot z} \quad (5.4)$$

$$A_W^* = \frac{A_W}{A_s} = \cos \alpha_n \quad (5.2)$$

### Deviations

$$f_b = d_b \cdot \frac{f_{Ha}}{L_a} \quad (6.2)$$

$$f_b = \frac{d_b}{m \cdot \pi \cdot \cos \alpha} \cdot f_{pe} = \frac{z}{\pi} \cdot f_{pe} \quad (6.6)$$

$$f_e = \frac{1}{2} \cdot \sqrt{f_{eV}^2 + f_{eR}^2 + 2 \cdot f_{eV} \cdot f_{eR} \cdot \cos \varphi_e} \quad (6.10)$$

$$f_{px} = -f_\beta \cdot \frac{p_x}{\sin |\beta| \cdot \cos \beta} \quad (6.8)$$

$$f_{pz} = -f_\beta \cdot \frac{p_z}{\sin |\beta| \cdot \cos \beta} \quad (6.9)$$

$$f_\alpha = -\frac{f_{Ha}}{L_a \cdot \tan \alpha_t} \quad (6.3)$$

$$f_\alpha = -\frac{f_{pe}}{m \cdot \pi \cdot \sin \alpha} \quad (6.5)$$

$$f_\beta \approx \frac{f_{HB}}{L_\beta} \cdot \cos^2 \beta \quad (6.7)$$

$$F_{pk} = \sum_k f_p \quad (6.1)$$

$$F_G \approx \tan F_G = \frac{1}{L} \cdot \sqrt{f_{eV}^2 + f_{eR}^2 - 2 \cdot f_{eV} \cdot f_{eR} \cdot \cos \varphi_e} \quad (6.11)$$

### Centre distances

$$a = \frac{1}{2} \cdot (d_{w1} + d_{w2}) \quad (3.3)$$

$$a = a_d \cdot \frac{\cos \alpha_t}{\cos \alpha_{wt}} = \frac{m_n \cdot (z_1 + z_2)}{2 \cdot \cos \beta} \cdot \frac{\cos \alpha_t}{\cos \alpha_{wt}} \quad (3.15)$$

$$a_d = \frac{d_1 + d_2}{2} = m_t \cdot \frac{z_1 + z_2}{2} = m_n \cdot \frac{z_1 + z_2}{2 \cdot \cos \beta} \quad (3.10)$$

$$a'' = \frac{(z_1 + z_L) \cdot m_t}{2} \cdot \frac{\cos \alpha_t}{\cos \alpha''} \quad (2.111)$$

### Diameters

$$d = z \cdot m_t = \frac{z \cdot m_n}{\cos \beta} \quad (2.4)$$

$$d_{a1} = 2 \cdot a - d_{f2} - 2 \cdot c = d_1 + 2 \cdot x_1 \cdot m_n + 2 \cdot h_{aP} + 2 \cdot k \cdot m_n \quad (2.36)$$

$$d_{a2} = 2 \cdot a - d_{f1} - 2 \cdot c = d_2 + 2 \cdot x_2 \cdot m_n + 2 \cdot h_{aP} + 2 \cdot k \cdot m_n \quad (2.37)$$

$$d_{aM} = d + 2 \cdot x_E \cdot m_n + 2 \cdot h_{aP} \quad (2.115)$$

$$d_b = d \cdot \cos \alpha_t = z \cdot m_t \cdot \cos \alpha_t = \frac{z \cdot m_n \cdot \cos \alpha_t}{\cos \beta} = z \cdot m_n / \sqrt{\tan^2 \alpha_n + \cos^2 \beta} = z \cdot m_b \quad (2.5)$$

$$d_f = d - 2 \cdot h_f + A_{st} \cdot \cot \alpha_t \quad (2.40)$$

$$d_{f1} = d_1 - 2 \cdot h_f = d_1 + 2 \cdot x_1 \cdot m_n - 2 \cdot h_{fP} \quad (2.38)$$

$$d_{f2} = d_2 - 2 \cdot h_f = d_2 + 2 \cdot x_2 \cdot m_n - 2 \cdot h_{fP} \quad (2.39)$$

$$d_v = d + 2 \cdot x \cdot m_n = d \cdot \left(1 + 2 \cdot \frac{x}{z} \cdot \cos \beta\right) \quad (2.41)$$

$$d_{w1} = \frac{2 \cdot z_1}{z_1 + z_2} \cdot a = \frac{2 \cdot a}{u + 1} = d_1 \cdot \frac{\cos \alpha_t}{\cos \alpha_{wt}} = \frac{d_{b1}}{\cos \alpha_{wt}} \quad (3.4)$$

$$d_{w2} = \frac{2 \cdot z_2}{z_1 + z_2} \cdot a = \frac{2 \cdot a \cdot u}{u + 1} = d_2 \cdot \frac{\cos \alpha_t}{\cos \alpha_{wt}} = \frac{d_{b2}}{\cos \alpha_{wt}} \quad (3.5)$$

$$d_K = d \cdot \frac{\cos \alpha_t}{\cos \alpha_{kt}} = \frac{d_b}{\cos \alpha_{kt}} \quad (2.104)$$

$$d_M = \frac{d_b}{\cos \alpha_M} \quad (2.106)$$

$$d_M = \sqrt{d_b^2 + (W_k \cdot \cos \beta_b)^2} \cdot \frac{z}{|z|} \quad (2.95)$$

$$d_{Na} \approx d_a - 2 \cdot \varrho_{an} \cdot (1 - \sin \alpha_{an}) \quad (3.28)$$

$$d_{NF1} = \sqrt{\left(2 \cdot a \cdot \sin \alpha_{wt} - \frac{z_2}{|z_2|} \cdot \sqrt{d_{Na2}^2 - d_{b2}^2}\right)^2 + d_{b1}^2} \quad (3.29)$$

$$d_{NF2} = \frac{z_2}{|z_2|} \sqrt{\left(2 \cdot a \cdot \sin \alpha_{wt} - \sqrt{d_{Na1}^2 - d_{b1}^2}\right)^2 + d_{b2}^2} \quad (3.30)$$

$$d_{NFE} = \sqrt{\left[d \cdot \sin \alpha_t - 2 \cdot (h_{a0} - x_E \cdot m_n - \varrho_{a0} \cdot (1 - \sin \alpha_n)) / \sin \alpha_t\right]^2 + d_b^2} \quad (3.31)$$

### Contact paths

$$g_a = \varrho_{E1} - \varrho_{C1} = \frac{1}{2} \cdot \left(\sqrt{d_{a1}^2 - d_{b1}^2} - d_{b1} \cdot \tan \alpha_{wt}\right) \quad (3.27)$$

$$g_f = \varrho_{A2} - \varrho_{C2} = \frac{1}{2} \cdot \left(\frac{z_2}{|z_2|} \cdot \sqrt{d_{a2}^2 - d_{b2}^2} - d_{b2} \cdot \tan \alpha_{wt}\right) \quad (3.26)$$

$$g_\alpha = \frac{1}{2} \cdot \left[\sqrt{d_{a1}^2 - d_{b1}^2} + \frac{z_2}{|z_2|} \cdot \sqrt{d_{a2}^2 - d_{b2}^2} - (d_{b1} + d_{b2}) \cdot \tan \alpha_{wt}\right] \quad (3.24)$$

$$g_\alpha = \frac{1}{2} \cdot \left(\sqrt{d_{a1}^2 - d_{b1}^2} - d_{b1} \cdot \tan \alpha_t\right) + \frac{m_n \cdot (1 - x_1)}{\sin \alpha_t} \quad (3.25)$$

$$g_{\alpha y} = \pm (\varrho_{C1} - \varrho_{y1}) = \mp (\varrho_{C2} - \varrho_{y2}) \quad (3.43)$$

Contact angles: see profile angles  
Virtual number of teeth

$$z_{nx} = \frac{z}{\cos^2 \beta_b \cdot \cos \beta} = z \cdot z_{nx}^* \quad (2.80)$$

$$z_{nM} = \frac{z}{\cos^{3,3} \beta} = z \cdot z_{nM}^* \quad (2.100)$$

$$z_{nW} = z \cdot \frac{\text{inv } \alpha_t}{\text{inv } \alpha_n} = z \cdot z_{nW}^* \quad (2.97)$$

Involute function

$$\text{inv } \alpha_{yt} = \xi_y - \alpha_{yt} = \tan \alpha_{yt} - \alpha_{yt} \quad (2.13)$$

Backlash

$$j_n = j_t \cdot \cos \alpha_n \cdot \cos \beta = j_t \cdot \cos \alpha_t \cdot \cos \beta_b \quad (9.1)$$

$$j_t = \frac{j}{2 \cdot \tan \alpha_{wt}} \quad (9.2)$$

Sliding factors

$$K_g = \frac{v_g}{v_t} = \frac{2 \cdot g_{ay}}{d_{w1}} \cdot \left(1 + \frac{1}{u}\right) \quad (3.47)$$

$$K_{ga} = \frac{2 \cdot g_a}{d_{w1}} \cdot \left(1 + \frac{1}{u}\right) \quad (3.49)$$

$$K_{gf} = \frac{2 \cdot g_f}{d_{w1}} \cdot \left(1 + \frac{1}{u}\right) \quad (3.48)$$

Sliding velocities

$$v_g = \pm \omega_1 \cdot \left(\frac{e_{y2}}{u} - e_{y1}\right) \quad (3.42)$$

$$v_g = \pm \omega_1 \cdot g_{ay} \cdot \left(1 + \frac{1}{u}\right) \quad (3.44)$$

$$v_{ga} = \pm \omega_1 \cdot g_a \cdot \left(1 + \frac{1}{u}\right) \quad (3.46)$$

$$v_{gf} = \pm \omega_1 \cdot g_f \cdot \left(1 + \frac{1}{u}\right) \quad (3.45)$$

Height: see tooth heights

Heights above the chords

$$\bar{h}_a \approx h_a + \frac{1}{4} \cdot d \cdot \psi^2 \cdot \cos^4 \beta \quad (2.87)$$

$$\bar{h}_c = h_a - \frac{s}{2} \cdot \cos \alpha \cdot \sin \alpha \quad (2.91)$$

$$\bar{h}_v \approx h_a - x \cdot m_n + \frac{1}{4} \cdot d_v \cdot \psi_v^2 \cdot \cos^4 \beta_v \quad (2.89)$$

$$\bar{h}_y \approx \frac{d_a - d_y}{2} + \frac{1}{4} \cdot d_y \cdot \psi_y^2 \cdot \cos^4 \beta_y \quad (2.88)$$

Addendum modification

$$k = y - \Sigma x \quad (3.17)$$

$$k \cdot m_n = a - a_d - m_n \cdot \Sigma x \quad (3.16)$$

Bottom clearance

$$c = h - h_w = c^* \cdot m_n \quad (3.9)$$

Radii of curvature of involutes

$$e_y = r_b \cdot \xi_y = r_b \cdot \tan \alpha_{yt} = \frac{z}{|z_2|} \cdot \sqrt{r_y^2 - r_b^2} \quad (2.12)$$

$$e_{A2} = \frac{1}{2} \cdot \frac{z_2}{|z_2|} \cdot \sqrt{d_{a2}^2 - d_{b2}^2} \quad (3.19)$$

$$e_{B1} = e_{E1} - p_{et} \quad (3.21)$$

$$e_{C1} = \frac{1}{2} \cdot \sqrt{d_{w1}^2 - d_{b1}^2} = \frac{1}{2} \cdot d_{b1} \cdot \tan \alpha_{wt} \quad (3.18)$$

$$e_{C1} + e_{C2} = a \cdot \sin \alpha_{wt} = e_{A1} + e_{A2} = e_{E1} + e_{E2} \quad (3.23)$$

$$e_{D2} = e_{A2} - p_{et} \quad (3.22)$$

$$e_{E1} = \frac{1}{2} \cdot \sqrt{d_{a1}^2 - d_{b1}^2} \quad (3.20)$$

Ball dimension: see dimension above  
balls or rollers  
Gap widths

$$e_{bn} = e_{bt} \cdot \cos \beta_b \quad (2.76)$$

$$e_{bt} = d_b \cdot \left(\frac{\pi - 4 \cdot x \cdot \tan \alpha_n}{2 \cdot z} - \text{inv } \alpha_t\right) \\ = \frac{e_{bn}}{\cos \beta_b} \quad (2.64, 2.76)$$

$$e_n = e_t \cdot \cos \beta = \frac{p_n}{2} - 2 \cdot x \cdot m_n \cdot \tan \alpha_n \\ = m_n \cdot \left(\frac{\pi}{2} - 2 \cdot x \cdot \tan \alpha_n\right) \quad (2.73)$$

$$e_t = \frac{p_t}{2} - 2 \cdot x \cdot m_n \cdot \tan \alpha_t \\ = m_t \cdot \left(\frac{\pi}{2} - 2 \cdot x \cdot \tan \alpha_n\right) = \frac{e_n}{\cos \beta} \quad (2.58, 2.73)$$

$$e_{vn} = e_{vt} \cdot \cos \beta_v \quad (2.75)$$

$$e_{vt} = d_v \cdot \left(\frac{e_t}{d} - \text{inv } \alpha_t + \text{inv } \alpha_{vt}\right) \\ = d_v \cdot \left(\frac{\pi - 4 \cdot x \cdot \tan \alpha_n}{2 \cdot z} - \text{inv } \alpha_t + \text{inv } \alpha_{vt}\right) \\ = \frac{e_{vn}}{\cos \beta_v} \quad (2.62, 2.75)$$

$$e_{yn} = e_{yt} \cdot \cos \beta_y \quad (2.74)$$

$$e_{yt} = d_y \cdot \left(\frac{e_t}{d} - \text{inv } \alpha_t + \text{inv } \alpha_{yt}\right) \\ = d_y \cdot \left(\frac{\pi - 4 \cdot x \cdot \tan \alpha_n}{2 \cdot z} - \text{inv } \alpha_t + \text{inv } \alpha_{yt}\right) \\ = \frac{e_{yn}}{\cos \beta_y} \quad (2.60, 2.74)$$

Dimension above balls or rollers

$$M_{dK} = d_K + D_M \quad (2.108)$$

$$M_{dK} = d_K \cdot \cos \frac{\pi}{2 \cdot z} + D_M \quad (2.109)$$

$$M_{dR} = \frac{M_{dK} - D_{MR}}{\cos \frac{\pi}{2 \cdot z}} + D_{MR} \quad (2.110)$$

$$M_{rK} = \frac{1}{2} \cdot (d_K + D_M) \quad (2.105)$$

**Diameter of measuring ball**

$$D_M \approx z_{nM} \cdot m_n \cdot \cos \alpha_n \cdot (\tan \alpha_{KnM} - \tan \alpha_{vnm}) \quad (2.99)$$

**Number of teeth measured**

$$k = \frac{z}{\pi} \cdot \left( \frac{\tan \alpha_{vt}}{\cos^2 \beta_b} - 2 \cdot \frac{x}{z} \cdot \tan \alpha_n - \text{inv } \alpha_t \right) + \frac{z}{2 \cdot |z|} \quad (2.92)$$

$$k \approx \frac{z_{nW}}{\pi} \cdot \left( \tan \alpha_{vn} - 2 \cdot \frac{x}{z_{nW}} \cdot \tan \alpha_n - \text{inv } \alpha_n \right) + \frac{z}{2 \cdot |z|} \quad (2.93)$$

$$k \approx z_{nW} \cdot \frac{\alpha_{vn}}{180} + \frac{z}{2 \cdot |z|} \quad (2.94)$$

**Modules**

$$m_b = m_n / \sqrt{\tan^2 \alpha_n + \cos^2 \beta} \quad (2.3)$$

$$m_n = m_t \cdot \cos \beta = m_x \cdot \sin |\beta| \quad (2.1, 2.2)$$

$$m_t = \frac{m_n}{\cos \beta} = m_x \cdot \tan |\beta| \quad (2.1, 2.2)$$

$$m_x = \frac{m_n}{\sin |\beta|} = \frac{m_n}{\cos \gamma} = \frac{m_t}{\tan |\beta|} \quad (2.2)$$

**Profile modification coefficients**

$$x_{\min} = -\frac{z \cdot \sin^2 \alpha_t}{2 \cdot \cos \beta} + \frac{h_{a0} - e_{a0} \cdot (1 - \sin \alpha_n)}{m_n} \quad (2.77)$$

$$x_E = x + \frac{A_s}{2 \cdot m_n \cdot \tan \alpha_n} \quad (2.114)$$

$$x_I'' = \frac{\frac{s_n + A_s}{m_n} - \frac{\pi}{2}}{2 \cdot \tan \alpha_n} \quad (2.113)$$

$$\Sigma x = \frac{(z_1 + z_2) \cdot (\text{inv } \alpha_{wt} - \text{inv } \alpha_t)}{2 \cdot \tan \alpha_n} \quad (3.14)$$

**Profile angles**

$$\cos \alpha = \frac{p_e M}{m \cdot \pi} \quad (6.4)$$

$$\tan \alpha_n = \tan \alpha_t \cdot \cos \beta \quad (2.9)$$

$$\cos \alpha_t = \frac{r_b}{r} = \frac{d_b}{d} = \frac{\cos \beta}{\cos \alpha_n} = \frac{\cos \alpha_n}{\sqrt{\tan^2 \alpha_n + \cos^2 \beta}} = \frac{\cos \alpha_n}{\sqrt{1 + \sin^2 \alpha_n \cdot \tan^2 \beta}} \quad (2.7)$$

$$\tan \alpha_t = \frac{\tan \alpha_n}{\cos \beta} \quad (2.9)$$

$$\cos \alpha_{vn} = \frac{\cos \alpha_n \cdot \cos \beta}{\cos \beta_v \cdot \left( 1 + 2 \cdot \frac{x}{z} \cdot \cos \beta \right)} = \frac{d}{d_v} \cdot \frac{\cos \alpha_n \cdot \cos \beta}{\cos \beta_v} \quad (2.45)$$

$$\cos \alpha_{vnm} = \frac{z_{nM} \cdot \cos \alpha_n}{z_{nM} + 2 \cdot x} \quad (2.101)$$

$$\tan \alpha_{vn} = \tan \alpha_{vt} \cdot \cos \beta_v \quad (2.44)$$

$$\cos \alpha_{vt} = \frac{z}{z + 2 \cdot x \cdot \cos \beta} \cdot \cos \alpha_t = \frac{\cos \alpha_t}{1 + 2 \cdot \frac{x}{z} \cdot \cos \beta} \quad (2.43)$$

$$\tan \alpha_{vt} = \frac{\tan \alpha_{vn}}{\cos \beta_v} \quad (2.44)$$

$$\cos \alpha_{wt} = \frac{d_{b1}}{d_{w1}} = \frac{d_{b2}}{d_{w2}} = \frac{(z_1 + z_2) \cdot m_t}{2 \cdot a} \cdot \cos \alpha_t \quad (3.6)$$

$$\text{inv } \alpha_{wt} = \text{inv } \alpha_t + 2 \cdot \frac{x_1 + x_2}{z_1 + z_2} \cdot \tan \alpha_n \quad (3.7)$$

$$\tan \alpha_{yn} = \tan \alpha_{yt} \cdot \cos \beta_y \quad (2.10)$$

$$\cos \alpha_{yt} = \frac{r_b}{r_y} = \frac{d_b}{d_y} = \frac{d}{d_y} \cdot \cos \alpha_t \quad (2.8)$$

$$\tan \alpha_{yt} = \frac{\tan \alpha_{yn}}{\cos \beta_y} \quad (2.10)$$

$$\tan \alpha_{yt} = \xi_y \quad (2.11)$$

$$\text{inv } \alpha_{yt} = \xi_y - \alpha_{yt} = \tan \alpha_{yt} - \alpha_{yt} \quad (2.13)$$

$$\text{inv } \alpha'' = \text{inv } \alpha_t + 2 \cdot \frac{x_1' + x_L}{z_1 + z_L} \cdot \tan \alpha_n \quad (2.112)$$

$$\alpha_{KnM} = \tan \alpha_{vnm} - \text{inv } \alpha_n + \frac{\pi - 4 \cdot x \cdot \tan \alpha_n}{2 \cdot z_{nM}} \quad (2.102)$$

$$\text{inv } \alpha_{Kt} = \frac{D_M}{z \cdot m_n \cdot \cos \alpha_n} - \eta + \text{inv } \alpha_t \quad (2.103)$$

$$= \frac{D_M}{d_b \cdot \cos \beta_b} - \eta_b \quad (2.103)$$

$$\tan \alpha_M = \tan \alpha_{Kt} - \frac{D_M}{d_b} \cdot \cos \beta_b \quad (2.107)$$

**Test dimensions for the tooth thickness:** see dimensions above balls or rollers, face width, chordal tooth thickness

**Roller dimensions:** see dimension above balls or rollers Helix angles

$$|\beta| = 90^\circ - |\gamma| \quad (2.29)$$

$$|\beta_b| = 90^\circ - |\gamma_b| \quad (2.6)$$

$$\sin \beta = \frac{\sin \beta_b}{\cos \alpha_n} \quad (2.31)$$

$$\cos \beta = \frac{\tan \alpha_n}{\tan \alpha_t} \quad (2.9)$$

$$\tan \beta = \frac{\tan \beta_b}{\cos \alpha_t} = \frac{\tan \beta_y \cdot \cos \alpha_{yt}}{\cos \alpha_t} \quad (2.30, 2.33)$$

$$\sin \beta_b = \sin \beta \cdot \cos \alpha_n \quad (2.31)$$

$$\begin{aligned} \cos \beta_b &= \cos \beta \cdot \frac{\cos \alpha_n}{\cos \alpha_t} = \frac{\sin \alpha_n}{\sin \alpha_t} = \frac{\sin \alpha_{yn}}{\sin \alpha_{yt}} \\ &= \cos \alpha_n \cdot \sqrt{\tan^2 \alpha_n + \cos^2 \beta} \end{aligned} \quad (2.32)$$

$$\tan \beta_b = \tan \beta \cdot \cos \alpha_t = \tan \beta_y \cdot \cos \alpha_{yt} \quad (2.30, 2.33)$$

$$\begin{aligned} \tan \beta_v &= \frac{z + 2 \cdot x \cdot \cos \beta}{z} \cdot \tan \beta \\ &= \tan \beta + 2 \cdot \frac{x}{z} \cdot \sin \beta = \tan \beta \cdot \frac{d_v}{d} \end{aligned} \quad (2.42)$$

$$\sin \beta_y = \sin \beta \cdot \frac{\cos \alpha_n}{\cos \alpha_{yn}} = \frac{\sin \beta_b}{\cos \alpha_{yn}} \quad (2.34)$$

$$\cos \beta_y = \frac{\tan \alpha_{yn}}{\tan \alpha_{yt}} = \frac{\cos \alpha_{yt} \cdot \cos \beta_b}{\cos \alpha_{yn}} \quad (2.35)$$

$$\begin{aligned} \tan \beta_y &= \tan \beta \cdot \frac{d_y}{d} = \tan \beta \cdot \frac{\cos \alpha_t}{\cos \alpha_{yt}} \\ &= \tan \beta_b \cdot \frac{d_y}{d_b} = \frac{\tan \beta_b}{\cos \alpha_{yt}} \end{aligned} \quad (2.33)$$

### Fluctuations

$$R_s = s_{\max} - s_{\min} \quad (6.12)$$

$$R_{Md} = M_{d\max} - M_{d\min} \quad (6.14)$$

$$R_W = W_{k\max} - W_{k\min} \quad (6.13)$$

### Specific sliding

$$G_1 = 1 - \frac{q_{y2}}{u \cdot q_{y1}} \quad (3.50)$$

$$G_2 = 1 - \frac{u \cdot q_{y1}}{q_{y2}} \quad (3.51)$$

$$G_{f1} = 1 - \frac{q_{A2}}{u \cdot q_{A1}} \quad (3.52)$$

$$G_{f2} = 1 - \frac{u \cdot q_{E1}}{q_{E2}} \quad (3.53)$$

### Ratio

$$g_\beta = r \cdot \varphi_\beta = b \cdot \tan |\beta| \quad (3.38)$$

### Helix slope

$$p_z = \frac{|z| \cdot m_n \cdot \pi}{\sin |\beta|} = \frac{|z| \cdot m_t \cdot \pi}{\tan |\beta|} = |z| \cdot p_x \quad (2.14)$$

### Helix angle

$$|\gamma| = 90^\circ - |\beta| \quad (2.29)$$

$$|\gamma_b| = 90^\circ - |\beta_b| \quad (2.6)$$

### Reference circle clearance factor

$$y = \frac{z_1 + z_2}{2 \cdot \cos \beta} \cdot \left( \frac{\cos \alpha_t}{\cos \alpha_{wt}} - 1 \right) \quad (3.12)$$

$$y \cdot m_n = a - a_d \quad (3.11)$$

$$y \cdot (\operatorname{inv} \alpha_{wt} - \operatorname{inv} \alpha_t) = \Sigma x \cdot \left( \frac{\cos \alpha_t}{\cos \alpha_{wt}} - 1 \right) \cdot \tan \alpha_t \quad (3.13)$$

### Pitches

$$p_{bn} = p_{bt} \cdot \cos \beta_b = p_n \cdot \cos \alpha_n \quad (2.25)$$

$$\begin{aligned} p_{bt} &= r_b \cdot \tau = \frac{d_b \cdot \pi}{z} = \frac{d_b}{d} \cdot p_t \\ &= p_t \cdot \cos \alpha_t = \frac{p_{bn}}{\cos \beta_b} \end{aligned} \quad (2.24, 2.25)$$

$$p_{bt} = s_{bt} + e_{bt} \quad (2.63)$$

$$p_{en} = p_n \cdot \cos \alpha_n = p_{bn} \quad (2.27)$$

$$p_{et} = p_t \cdot \cos \alpha_t = p_{bt} \quad (2.26)$$

$$p_k = k \cdot p_t \quad (2.19)$$

$$p_n = p_t \cdot \cos \beta = m_n \cdot \pi \quad (2.18)$$

$$p_t = r \cdot \tau = \frac{d \cdot \pi}{z} = m_t \cdot \pi = \frac{m_n \cdot \pi}{\cos \beta} = \frac{p_n}{\cos \beta} \quad (2.17, 2.18)$$

$$p_t = s_t + e_t \quad (2.57)$$

$$p_{vn} = p_{vt} \cdot \cos \beta_v \quad (2.23)$$

$$p_{vt} = r_v \cdot \tau = \frac{d_v \cdot \pi}{z} = \frac{d_v}{d} \cdot p_t = \frac{p_{vn}}{\cos \beta_v} \quad (2.22, 2.23)$$

$$p_{vt} = s_{vt} + e_{vt} \quad (2.61)$$

$$p_x = m_x \cdot \pi = \frac{m_n \cdot \pi}{\sin |\beta|} = \frac{m_t \cdot \pi}{\tan |\beta|} = \frac{p_z}{|z|} \quad (2.28)$$

$$p_{yn} = p_{yt} \cdot \cos \beta_y \quad (2.21)$$

$$p_{yt} = r_y \cdot \tau = \frac{d_y \cdot \pi}{z} = \frac{d_y}{d} \cdot p_t = \frac{p_{yn}}{\cos \beta_y} \quad (2.20, 2.21)$$

$$p_{yt} = s_{yt} + e_{yt} \quad (2.59)$$

$$p_z = \frac{|z| \cdot m_n \cdot \pi}{\sin |\beta|} = \frac{|z| \cdot m_t \cdot \pi}{\tan |\beta|} = |z| \cdot p_x \quad (2.14)$$

### Pitch angles

$$\tau = \frac{2 \cdot \pi}{z} \text{ in Radian} \quad (2.15)$$

$$\tau = \frac{360}{z} \text{ in Grad} \quad (2.16)$$

### Tolerances

$$T_a = A_{ae} - A_{ai} \quad (8.1)$$

$$T_{a^*} = A_{a^*e} - A_{a^*i} \quad (4.10)$$

$$T_{da} = A_{dae} - A_{dai} \quad (4.11)$$

$$T_s = A_{se} - A_{si} \quad (4.5)$$

$$T_{\bar{s}} = A_{\bar{s}e} - A_{\bar{s}i} \quad (4.6)$$

$$T_{Md} = A_{Mdc} - A_{Mdi}$$

$$T_{Mr} = A_{Mrc} - A_{Mri}$$

$$T_W = A_{Wc} - A_{Wi}$$

### Overlaps

$$\epsilon_\alpha = \frac{\varphi_{\alpha 1}}{\tau_1} = \frac{\varphi_{\alpha 2}}{\tau_2} = \frac{g_\alpha}{p_{et}} = \frac{g_i + g_a}{p_{et}} \quad (3.34)$$

$$\epsilon_\beta = \frac{\varphi_{\beta 1}}{\tau_1} = \frac{\varphi_{\beta 2}}{\tau_2} = \frac{b}{p_x} = \frac{b \cdot \sin |\beta|}{m_n \cdot \pi} \quad (3.37)$$

$$\epsilon_\gamma = \frac{\varphi_{\gamma 1}}{\tau_1} = \frac{\varphi_{\gamma 2}}{\tau_2} = \epsilon_\alpha + \epsilon_\beta \quad (3.41)$$

### Overlap angles

$$\varphi_{\alpha 1} = \frac{g_\alpha}{r_{b1}} = u \cdot \varphi_{\alpha 2} \quad (3.32)$$

$$\varphi_{\alpha 2} = \frac{g_\alpha}{r_{b2}} = \frac{\varphi_{\alpha 1}}{u} \quad (3.33)$$

$$\varphi_{\beta 1} = \frac{b \cdot \tan |\beta|}{r_1} = \frac{2 \cdot b \cdot \sin |\beta|}{m_n \cdot z_1} = u \cdot \varphi_{\beta 2} \quad (3.35)$$

$$\varphi_{\beta 2} = \frac{b \cdot \tan |\beta|}{r_2} = \frac{2 \cdot b \cdot \sin |\beta|}{m_n \cdot z_2} = \frac{\varphi_{\beta 1}}{u} \quad (3.36)$$

$$\varphi_{\gamma 1} = \varphi_{\alpha 1} + \varphi_{\beta 1} = u \cdot \varphi_{\gamma 2} \quad (3.39)$$

$$\varphi_{\gamma 2} = \varphi_{\alpha 2} + \varphi_{\beta 2} = \frac{\varphi_{\gamma 1}}{u} \quad (3.40)$$

### Transmission ratio

$$i = \frac{\omega_a}{\omega_b} = \frac{n_a}{n_b} = -\frac{z_b}{z_a} \quad (3.2)$$

### Composite length

$$L_y = |r_b| \cdot \tan \alpha_{yt} = \sqrt{r_y^2 - r_b^2} \quad (2.12)$$

### Pitch angle of involute

$$\zeta_y = \tan \alpha_{yt} \quad (2.11)$$

### Tooth number ratio

$$u = \frac{z_2}{z_1} \quad (3.1)$$

### Facewidth

$$b = p_x \cdot \epsilon_\beta = \frac{m_n \cdot \pi \cdot \epsilon_\beta}{\sin |\beta|} \quad (3.37)$$

$$b = \frac{m_n \cdot z_1 \cdot \varphi_{\beta 1}}{2 \cdot \sin |\beta|} = \frac{m_n \cdot z_2 \cdot \varphi_{\beta 2}}{2 \cdot \sin |\beta|} \quad (3.35, 3.36)$$

$$b \geq W_k \cdot \sin |\beta_h| + b_M \cdot \cos \beta_b \quad (2.98)$$

### Tooth thickness

$$s_{bn} = s_{bt} \cdot \cos \beta_b \quad (2.72)$$

$$s_{bt} = d_b \cdot \left( \frac{\pi + 4 \cdot x \cdot \tan \alpha_n}{2 \cdot z} + \text{inv } \alpha_t \right) = \frac{s_{bn}}{\cos \beta_b} \quad (2.52, 2.72)$$

$$s_n = s_t \cdot \cos \beta = \frac{p_n}{2} + 2 \cdot x \cdot m_n \cdot \tan \alpha_n = m_n \cdot \left( \frac{\pi}{2} + 2 \cdot x \cdot \tan \alpha_n \right) \quad (2.69)$$

$$s_t = \frac{p_t}{2} + 2 \cdot x \cdot m_n \cdot \tan \alpha_t = m_t \cdot \left( \frac{\pi}{2} + 2 \cdot x \cdot \tan \alpha_n \right) = \frac{s_n}{\cos \beta} \quad (2.49, 2.69)$$

$$s_{vn} = s_{vt} \cdot \cos \beta_v \quad (2.71)$$

$$s_{vt} = d_v \cdot \left( \frac{s_t}{d} + \text{inv } \alpha_t - \text{inv } \alpha_{vt} \right) = d_v \cdot \left( \frac{\pi + 4 \cdot x \cdot \tan \alpha_n}{2 \cdot z} + \text{inv } \alpha_t - \text{inv } \alpha_{vt} \right) = \frac{s_{vn}}{\cos \beta_v} \quad (2.51, 2.71)$$

$$s_{yn} = s_{yt} \cdot \cos \beta_y \quad (2.70)$$

$$s_{yt} = d_y \cdot \left( \frac{s_t}{d} + \text{inv } \alpha_t - \text{inv } \alpha_{yt} \right) = d_y \cdot \left( \frac{\pi + 4 \cdot x \cdot \tan \alpha_n}{2 \cdot z} + \text{inv } \alpha_t - \text{inv } \alpha_{yt} \right) = \frac{s_{yn}}{\cos \beta_y} \quad (2.50, 2.70)$$

### Tooth thickness error

$$A_s = (d_{aM} - d_a) \cdot \tan \alpha_n \quad (2.116)$$

$$A_{st} = \frac{A_s}{\cos \beta} \quad (4.2)$$

$$A_{sy} = A_s \cdot \frac{d_y \cdot \cos \beta_y}{d \cdot \cos \beta} \approx A_s \cdot \left( 1 + 2 \cdot \frac{x}{z_{nW}} \right) \quad (4.1)$$

$$A_{Mt} \approx \frac{1}{2} \cdot \frac{\cos \alpha_t}{\sin \alpha_{kt}} \cdot \frac{A_s}{\cos \beta} \quad (4.4)$$

### Tooth thickness half angle

$$\psi = \frac{s_t}{d} = \frac{\pi + 4 \cdot x \cdot \tan \alpha_n}{2 \cdot z} \quad (2.53)$$

$$\psi_a = \frac{s_{at}}{d_a} = \psi + \text{inv } \alpha_t - \text{inv } \alpha_{at} \quad (2.78)$$

$$\psi_b = \frac{s_{bt}}{d_b} = \psi + \text{inv } \alpha_t \quad (2.56)$$

$$\psi_v = \frac{s_{vt}}{d_v} = \psi + \text{inv } \alpha_t - \text{inv } \alpha_{vt} \quad (2.55)$$

$$\psi_y = \frac{s_{yt}}{d_y} = \psi + \text{inv } \alpha_t - \text{inv } \alpha_{yt} \quad (2.54)$$

**hordal tooth thicknesses**

$$\bar{s}_c = s \cdot \cos^2 \alpha = m \cdot \cos^2 \alpha \cdot \left( \frac{\pi}{2} + 2 \cdot x \cdot \tan \alpha \right) \quad (2.90)$$

$$\bar{s}_n = d_n \cdot \sin \psi_n = \frac{d \cdot \sin (\psi \cdot \cos^3 \beta)}{\cos^2 \beta} \quad (2.81)$$

$$\bar{s}_n \approx s_n \cdot \left( 1 - \frac{1}{6} \cdot \psi^2 \cdot \cos^6 \beta \right) \quad (2.84)$$

$$\bar{s}_{vn} = \frac{d_v \cdot \sin (\psi_v \cdot \cos^3 \beta_v)}{\cos^2 \beta_v} \quad (2.83)$$

$$\bar{s}_{vn} \approx s_{vn} \cdot \left( 1 - \frac{1}{6} \cdot \psi_v^2 \cdot \cos^6 \beta_v \right) \quad (2.86)$$

$$\bar{s}_{yn} = \frac{d_y \cdot \sin (\psi_y \cdot \cos^3 \beta_y)}{\cos^2 \beta_y} \quad (2.82)$$

$$\bar{s}_{yn} \approx s_{yn} \cdot \left( 1 - \frac{1}{6} \cdot \psi_y^2 \cdot \cos^6 \beta_y \right) \quad (2.85)$$

**Tooth depths**

$$h = h_p + k \cdot m_n \quad (2.46)$$

$$h_a = h_{ap} + x \cdot m_n + k \cdot m_n \quad (2.47)$$

$$h_f = h_{fp} - x \cdot m_n \quad (2.48)$$

$$h_w = \frac{d_{a1} + d_{a2}}{2} - a \quad (3.8)$$

**Tooth gaps: see widths  
Space widths half angles**

$$\eta = \frac{e_t}{d} = \frac{\pi - 4 \cdot x \cdot \tan \alpha_n}{2 \cdot z} \quad (2.65)$$

$$\eta = \frac{e_t - A_{st}}{d} = \frac{\pi - 4 \cdot x \cdot \tan \alpha_n}{2 \cdot z} - \frac{A_s}{z \cdot m_n} \quad (4.3)$$

$$\eta_b = \frac{e_{bt}}{d_b} = \eta - \text{inv } \alpha_t \quad (2.68)$$

$$\eta_f = \frac{e_{ft}}{d_f} = \eta - \text{inv } \alpha_t + \text{inv } \alpha_{ft} \quad (2.79)$$

$$\eta_v = \frac{e_{vt}}{d_v} = \eta - \text{inv } \alpha_t + \text{inv } \alpha_{vt} \quad (2.67)$$

$$\eta_y = \frac{e_{yt}}{d_y} = \eta - \text{inv } \alpha_t + \text{inv } \alpha_{yt} \quad (2.66)$$

**Base tangent length**

$$W_k = m_n \cdot \cos \alpha_n \cdot \left[ \left( k - \frac{z}{2 \cdot |z|} \right) \cdot \pi + z \cdot \text{inv } \alpha_t \right] + 2 \cdot x \cdot m_n \cdot \sin \alpha_n \quad (2.96)$$

3 INDEX  
Omitted.